

Features of de Sitter vacua in M-theory

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ABSTRACT: We compute the masses of all moduli in the unstable de Sitter vacua arising in the toy model of cosmological M-theory flux compactifications on the G_2 holonomy manifolds of [1]. The slow-roll parameters in the tachyonic directions are shown to be too large to be useful for conventional models of inflation. However, it appears that we can find fast roll regimes which could, under certain conditions, account for the current dark energy driven accelerated expansion of the universe.

KEYWORDS: dS vacua in string theory, M-Theory.

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1. Introduction

There is a long history of studying de Sitter vacua in string theory. Early development included the study of [2] which identified de Sitter space as a solution in the gauged extended supergravities that arise as consistent truncations of 10-dimensional type-II supergravity and 11-dimensional supergravity in backgrounds of the form $dS_4 \times H^{p,q}$ where the second factor is a hyperbolic space (of six or seven dimensions). These backgrounds are non-compact and hence the gauging is a noncompact one. Other studies of these ghost-free supergravities included [4, 5].

The study of de Sitter space in a stable compactification of type-II string theory was initiated by KKLT [6]. A mechanism of perturbative stabilization of M-theory vacua was proposed in [7] by turning on flux on the internal space as well as turning on flux on singular loci within the internal space. In another line of investigation, moduli stabilization was achieved by non-perturbative contributions to the superpotential following work on determining more precise expressions for the Kähler potential in compactifications on G_2 manifolds realized as blowups of singular seven tori [8–10].

In [1] the authors were interested in the statistics of vacua of M-theory on manifolds of G_2 holonomy. They used a toy model which is similar to the one discussed in [7], where a perturbative Chern-Simons contribution is necessary to stabilize the moduli. The model shares the feature of the pre-KKLT models in that its de Sitter vacua have tachyonic directions. We will attempt to take this model more seriously by computing the cosmological parameters that they produce.

It is widely accepted that the universe (or more precisely our Hubble volume) has undergone two periods of accelerated expansion in its history. The first of these is inflation [11], which took place in some earlier epoch, and the second is the dark energy driven

expansion that we observe today (see [12] for recent data). Both of these accelerated periods can result from slowly-rolling scalar fields. Observational constraints can be placed on the parameters ε, η that quantify the degree of slow roll allowing us to assess the usefulness of this model for cosmological purposes.

The scalar field potential we will obtain actually gives a somewhat more involved model than is often used. The de Sitter vacuum and associated tachyonic directions will give accelerated expansion. However, such expansion will only continue until the potential runs negative, resulting in the collapse of the universe. In this case our concern (for dark energy at least) is that the potential is flat enough and that the field starts close enough to the maximum so as to be consistent with the age of the observable universe. Such models are discussed in detail in [2, 3] and [13].

In another interesting direction, it was noted in [14] that the examples of de Sitter space from gauged supergravity truncations of type-II and 11-dimensional supergravity had the strange property that the scalar mass spectrum was quantized in units of $1/3$ of the cosmological constant. It was shown that the mass of a scalar was a particular Casimir in the algebra of isometries of dS_4 but there was no reason for that Casimir to be quantized. The list of examples of de Sitter vacua with this behavior was quite impressive, including all examples from gauged $\mathcal{N} = 2, 4$ and 8 supergravity studied in [2]. The feature was found to persist in models with tachyon-free dS vacua [15]. It turns out that the M-theory models of [1], with appropriate fluxes, generate scalar masses that may or may not have quantized scalars.

2. The model

We are interested in a compactification of M-theory on a manifold X of G_2 holonomy. We use the conventions of [1] which we will summarize. The resulting theory in four dimensions is an $\mathcal{N} = 1$ supergravity specified by the field content, Kähler potential and superpotential. The fields are, as usual, the complexified coordinates on the moduli space, \mathcal{M} , of metric deformations on X . This moduli space has dimension $n = b^3(X)$ and has complexified coordinates given by the periods of the G_2 invariant three-form Φ complexified by the periods of the three-form potential C_3 ,

$$z^i = t^i + i s^i = \frac{1}{l_M^3} \int (C_3 + i\Phi) . \tag{2.1}$$

The moduli space of metrics is Kähler with Kähler potential given by

$$K(z, \bar{z}) = -3 \ln(4\pi^{\frac{1}{3}} V_X(s)) . \tag{2.2}$$

There are no known strong constraints on the dependence of the volume of the manifold X on the moduli s except that it be a homogeneous function of degree $7/3$ and that the above constructed K is convex. Thus the most general function $V_X(s)$ is

$$V_X(s) = \prod_{k=1}^n s_k^{a_k} f(s) \tag{2.3}$$

with the exponents a_k such that

$$\sum_{k=1}^n a_k = \frac{7}{3}. \tag{2.4}$$

The arbitrary function $f(s)$ is invariant under rescaling and thus, to leading order, can be taken to be one

$$V_X(s) = \prod_{k=1}^n s_k^{a_k}. \tag{2.5}$$

As usual, we turn on an internal flux of G_4 which induces a GVW superpotential [16]

$$W \sim \int_X (C_3 + i\Phi) \wedge G_4. \tag{2.6}$$

As described in [7], we'll actually need more than this to stabilize the moduli. Consider G_2 manifolds X that have an ADE orbifold singularity. The center of the ADE space is a singular (and supersymmetric) three-cycle, Q . There are light M2 branes localized at the singular three-cycle due to the shrinking two-cycles of the ADE space. These light degrees of freedom combine to make a Chern-Simons gauge theory on Q with gauge group the complexification of the ADE gauge group, $G^{\mathbb{C}}$. The Chern-Simons background is determined by a flat connection which makes a complex constant contribution to the superpotential. The final superpotential is thus given by

$$W(z) = \frac{1}{\kappa_4^3} (N_i z^i + c_1 + i c_2). \tag{2.7}$$

The constant contribution c_2 will turn out to be a crucial ingredient. Although there are no known explicit examples of how to compute these constants, there is no good reason to forbid their existence.

3. de Sitter critical points

Given the above expressions for the Kähler and superpotential, the potential can be computed as usual,

$$V(z, \bar{z}) = \kappa_4^2 e^K (g^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2) \tag{3.1}$$

which yields the following [1]:

$$V = \frac{c_2^2}{48\pi V_X^3} \left(3 + \sum_{j=1}^n a_j \nu_j s_j (\nu_j s_j - 3) \right) + \frac{1}{48\pi V_X^3} \left(\vec{N} \cdot \vec{t} + c_1 \right)^2 \tag{3.2}$$

where $\nu_i \equiv -\frac{N_i}{c_2 a_i}$. In the single modulus case (dropping the axion term), we have:

$$V = \frac{1}{16\pi} \left(\frac{c_2^2}{s^7} + \frac{N c_2}{s^6} + \frac{N^2}{7s^5} \right) \tag{3.3}$$

In order to calculate slow roll parameters we need to use canonically-normalized scalar fields. The Kähler potential in the previous section leads to the following Kähler metric:

$$ds^2 = \sum_{i=1}^n \frac{3a_i}{4s_i^2} dz_i d\bar{z}_i = \sum_{i=1}^n (dt_i^2 + ds_i^2) \quad (3.4)$$

This will give a kinetic term that looks like:

$$g_{i\bar{j}} \partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} = \sum_i \frac{3a_i}{4s_i^2} (\partial_\mu t_i \partial^\mu t_i + \partial_\mu s_i \partial^\mu s_i) \quad (3.5)$$

Ignoring the t -terms we see that if we define:

$$s_i = \exp\left(\sqrt{\frac{2}{3a_i}} \phi_i\right) \quad (3.6)$$

Then, ϕ_i are the canonically normalized moduli fields. For our single modulus case we will have $s = \exp\left(\sqrt{\frac{2}{7}} \phi\right)$, which then gives the potential as:

$$V(\phi) = \frac{1}{16\pi} \left(c_2^2 e^{(-7\sqrt{\frac{2}{7}})\phi} + N c_2 e^{(-6\sqrt{\frac{2}{7}})\phi} + \frac{N^2}{7} e^{(-5\sqrt{\frac{2}{7}})\phi} \right) \quad (3.7)$$

We then straightforwardly obtain:

$$V'(\phi) = -\frac{1}{16\pi} \sqrt{\frac{2}{7}} \left(7c_2^2 e^{(-7\sqrt{\frac{2}{7}})\phi} + 6N c_2 e^{(-6\sqrt{\frac{2}{7}})\phi} + \frac{5N^2}{7} e^{(-5\sqrt{\frac{2}{7}})\phi} \right) \quad (3.8)$$

$$V''(\phi) = \frac{1}{56\pi} \left(49c_2^2 e^{(-7\sqrt{\frac{2}{7}})\phi} + 36N c_2 e^{(-6\sqrt{\frac{2}{7}})\phi} + \frac{25N^2}{7} e^{(-5\sqrt{\frac{2}{7}})\phi} \right) \quad (3.9)$$

We can set $V' = 0$ and find the critical points. The unstable de Sitter maxima corresponds to $s = -\frac{7c_2}{N}$. The slow roll parameters are defined as:

$$\varepsilon = \frac{1}{2} \left(\frac{V'}{V} \right)^2 \quad (3.10)$$

$$\eta = \frac{V''}{V} \quad (3.11)$$

To compute these parameters around the critical point, $\tilde{\phi} = \sqrt{\frac{7}{2}} \ln\left(\frac{-7c_2}{N}\right)$, we set $\phi = \tilde{\phi} + \delta$ and eventually obtain:

$$\varepsilon = \frac{28 \left(-3 + 3 \cosh \left[\sqrt{\frac{2}{7}} \delta \right] + 2 \sinh \left[\sqrt{\frac{2}{7}} \delta \right] \right)^2}{\left(-7 + 8 \cosh \left[\sqrt{\frac{2}{7}} \delta \right] + 6 \sinh \left[\sqrt{\frac{2}{7}} \delta \right] \right)^2} \quad (3.12)$$

$$\eta = \frac{50}{7} + \frac{48 - 154e^{\sqrt{\frac{2}{7}}\delta}}{7 - 49e^{\sqrt{\frac{2}{7}}\delta} + 49e^{2\sqrt{\frac{2}{7}}\delta}} \quad (3.13)$$

Expanding around $\delta = 0$ gives:

$$\varepsilon = 32\delta^2 - 48\sqrt{14}\delta^3 + O[\delta^4] \tag{3.14}$$

$$\eta = -8 + 12\sqrt{14}\delta - \frac{880}{7}\delta^2 + \frac{4636}{7}\sqrt{\frac{2}{7}}\delta^3 + O[\delta^4] \tag{3.15}$$

These are neither small, nor obviously tunable — since they are independent of fluxes.

Using the results and machinery of [1] we can generalise this to n moduli fields. First, note that with the canonically-normalized $s_i = \exp\sqrt{\frac{2}{3a_i}}\phi_i$ and our earlier expression for V (3.2), we will get:

$$\frac{\partial V}{\partial \phi_i} = \sqrt{\frac{2a_i}{3}} \frac{c_2^2}{48\pi V_X^3} (-3E + 2\nu_i^2 s_i^2 - 3\nu_i s_i) \tag{3.16}$$

$$\begin{aligned} \frac{\partial^2 V}{\partial \phi_j \partial \phi_i} &= \frac{2}{3} \frac{c_2^2}{48\pi V_X^3} \times \\ &\times (3\sqrt{a_i a_j} (3E - 2(\nu_i^2 s_i^2 + \nu_j^2 s_j^2) + 3(\nu_i s_i + \nu_j s_j)) + \delta_{ij} (4\nu_i^2 s_i^2 - 3\nu_i s_i)) \end{aligned} \tag{3.17}$$

E is defined as

$$E = 3 + \sum_{j=1}^n a_j \nu_j s_j (\nu_j s_j - 3) \tag{3.18}$$

The critical points are obtained in [1]. If we define $h_i = \nu_i s_i$, then the solutions are:

$$h_i = \frac{3}{4} + m_i H \tag{3.19}$$

With $m_i = \pm 1$, $A = \vec{a} \cdot \vec{m}$ and H given by:

$$H(A) = \frac{3}{20} (3A - \sqrt{9A^2 + 15}) \tag{3.20}$$

We'll rewrite the η matrix at a critical point determined by the m_i as follows

$$\eta_{ij}(a_i, m_i) = Q(A)\sqrt{a_i a_j} + S(A)\delta_{ij} + T(A) \left(\frac{1 - m_j}{2} \right) \delta_{ij} \tag{3.21}$$

where

$$\begin{aligned} Q(A) &= \frac{2}{E(A)} (3E(A) + 9/4 - 4H(A)^2) \\ S(A) &= \frac{2H(A)}{E(A)} \left(\frac{4H(A)}{3} + 1 \right) \\ T(A) &= \frac{-4H(A)}{E(A)} \\ E(A) &= \frac{-15}{16} - \frac{3}{2}AH(A) + \frac{7}{3}H(A)^2 \end{aligned} \tag{3.22}$$

As pointed out by Acharya et al., the physical vacua can be put into three categories determined by requiring the moduli fields to be positive. When all fluxes are negative, there are several AdS minima with $A > -1/3$ as well as one dS maximum with $A = -7/3$ when all $m_i = -1$. When some of the fluxes are not negative and $A < -1/3$ then there is a single physical vacuum which is a dS maximum when $sign(N_i) = sign(m_i)$. When some of the fluxes are negative and $A > -1/3$ then there are no vacua that have positive moduli. We will now consider the slow roll parameters of the dS vacua in the two cases when they occur.

3.1 All negative fluxes

For negative fluxes, the dS vacuum is at all $m_i = -1$. This gives for η

$$\eta_{ij}(a_i, m_i = -1) = V^{-1} \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} = 6(\delta_{ij} - \sqrt{a_i a_j}) . \quad (3.23)$$

We are interested in eigenvalues of this matrix. Define $\alpha_i = \sqrt{a_i}$. Note that α_i is a n-vector with norm $\sum_{i=1}^n \alpha_i \alpha_i = 7/3$. Then the matrix is

$$\eta_{ij} = 6(\delta_{ij} - \alpha_i \alpha_j) \quad (3.24)$$

So first, it is clear that α_i is an eigenvector of η with eigenvalue -8. Since α_i is a nonzero n-vector, there are n-1 orthogonal vectors. Each such orthogonal vector is in fact an eigenvector of η with eigenvalue 6. Hence the eigenvalues of η are -8 and 6 with multiplicity 1 and n-1. Needless to say, this is independent of what values the a_i take as long as they satisfy the constraint $\sum_i a_i = 7/3$.

It was discussed in [14] that the Casimir operator in dS_4 space, $k = \frac{3m^2}{\Lambda} = \frac{m^2}{H^2} = 3\eta$, in all known extended supergravities takes integer values. There is no known explanation of this observation. Here we find again that $3m^2/\Lambda = -24, 18$ are integral.

3.2 Some negative fluxes

To make some progress in this more general case let us consider again the form of the η matrix:

$$\eta_{ij}(a_i, m_i) = Q(A)\sqrt{a_i a_j} + S(A)\delta_{ij} + T(A) \left(\frac{1 - m_j}{2} \right) \delta_{ij} \quad (3.25)$$

where

$$\begin{aligned} Q(A) &= \frac{2}{E(A)} \left(3E(A) + \frac{9}{4} - 4H(A)^2 \right) \\ S(A) &= \frac{2H(A)}{E(A)} \left(\frac{4H(A)}{3} + 1 \right) \\ T(A) &= \frac{-4H(A)}{E(A)} \\ E(A) &= \frac{-15}{16} - \frac{3}{2}AH(A) + \frac{7}{3}H(A)^2 \end{aligned} \quad (3.26)$$

Let's consider the case when we have k negative fluxes and the rest positive. Since we are free to reorder the a_i we can, without loss of generality, set N_1 through to N_k to be negative, while N_{k+1} through N_n are positive. This then gives:

$$m_i = \text{sign}(N_i) = \begin{cases} -1 & \text{if } i = 1 \dots k \\ 1 & \text{if } i = k + 1 \dots n \end{cases} \quad (3.27)$$

With this in mind we then write η as:

$$\eta = \begin{pmatrix} M^- & q \\ q^T & M^+ \end{pmatrix} \quad (3.28)$$

where

$$M^- = Q(A)\sqrt{a_i a_j} + (S(A) + T(A))\delta_{ij} \quad (i, j = 1, \dots, k) \quad (3.29)$$

$$M^+ = Q(A)\sqrt{a_i a_j} + S(A)\delta_{ij} \quad (i, j = k + 1, \dots, n) \quad (3.30)$$

$$q = Q(A)\sqrt{a_i a_j} \quad (i = 1, \dots, k; j = k + 1, \dots, n) \quad (3.31)$$

Now we set $\alpha_i = \sqrt{a_i}$ and define the following:

$$v_i^- = \begin{cases} \alpha_i & \text{if } i = 1 \dots k \\ 0 & \text{if } i = k + 1 \dots n \end{cases} \quad (3.32)$$

$$v_i^+ = \begin{cases} 0 & \text{if } i = 1 \dots k \\ \alpha_i & \text{if } i = k + 1 \dots n \end{cases} \quad (3.33)$$

$$w_i^- = \begin{cases} w_i & \text{if } i = 1 \dots k \\ 0 & \text{if } i = k + 1 \dots n \end{cases} \quad (3.34)$$

$$w_i^+ = \begin{cases} 0 & \text{if } i = 1 \dots k \\ w_i & \text{if } i = k + 1 \dots n \end{cases} \quad (3.35)$$

We choose w^\pm such that $\alpha \cdot w^\pm = 0$. This gives us $k - 1$ choices for w^- and $n - k - 1$ choices for w^+ , a point we shall return to momentarily. It is reasonably straightforward to show that w^\pm are eigenvectors of η :

$$\begin{aligned} \eta w^- &= \begin{pmatrix} M^- & q \\ q^T & M^+ \end{pmatrix} \begin{pmatrix} w_i^- \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} M_{ij}^- w_j^- \\ q_{ij}^T w_j^- \end{pmatrix} \\ &= \begin{pmatrix} (Q(A)\alpha_i \alpha_j + (S(A) + T(A))\delta_{ij}) w_j^- \\ Q(A)\alpha_i \alpha_j w_j^- \end{pmatrix} \\ &= \begin{pmatrix} (S(A) + T(A)) w_i^- \\ 0 \end{pmatrix} \\ &= (S(A) + T(A))w^- \end{aligned} \quad (3.36)$$

In the final step we used that fact that $\alpha \cdot w^- = 0$ and that w_i is equal to zero for $i = k + 1 \dots n$. Thus the $k - 1$ w^- are eigenvectors with $S(A) + T(A)$ the associated eigenvalue of multiplicity $k - 1$. An analogous argument follows for w^+ , this time giving an eigenvalue of $S(A)$ with a multiplicity of $n - k - 1$.

We have found all but two of our eigenvalues. Sadly, though, things are not quite so straightforward for the final two. Let us consider the action of η on v^\pm :

$$\eta v^- = \begin{pmatrix} M^- & q \\ q^T & M^+ \end{pmatrix} \begin{pmatrix} v_i^- \\ 0 \end{pmatrix}$$

$$\begin{aligned}
&= \begin{pmatrix} M_{ij}^- v_j^- \\ q_{ij}^T v_j^- \end{pmatrix} \\
&= \begin{pmatrix} (Q(A)\alpha_i\alpha_j + (S(A) + T(A))\delta_{ij})\alpha_j \\ Q(A)\alpha_i\alpha_j\alpha_j \end{pmatrix} \\
&= \begin{pmatrix} (|\alpha_k^+|^2 Q(A) + (S(A) + T(A)))\alpha_i \\ |\alpha_k^+|^2 Q(A)\alpha_i \end{pmatrix} \\
&= |\alpha_k^-|^2 Q(A)(v^- + v^+) + (S(A) + T(A))v^- \quad |\alpha_k^-|^2 = \alpha_1^2 + \dots + \alpha_k^2 \quad (3.37)
\end{aligned}$$

And, in a similar fashion, we obtain:

$$\eta v^+ = |\alpha_k^+|^2 Q(A)(v^- + v^+) + S(A)v^+ \quad |\alpha_k^+|^2 = \alpha_{k+1}^2 + \dots + \alpha_n^2 \quad (3.38)$$

Since v^\pm are sent to linear combinations of v^\pm by the action of η we know that the remaining two eigenvectors are themselves linear combinations of v^\pm . Thus we can find the remaining eigenvalues by diagonalizing the matrix:

$$\begin{pmatrix} |\alpha_k^-|^2 Q(A) + S(A) + T(A) & |\alpha_k^+|^2 Q(A) \\ |\alpha_k^-|^2 Q(A) & |\alpha_k^+|^2 Q(A) + S(A) \end{pmatrix} \quad (3.39)$$

These eigenvalues are:

$$\frac{1}{2} \left(2S + T + Q (|\alpha_k^-|^2 + |\alpha_k^+|^2) \pm \sqrt{T^2 + 2QT (|\alpha_k^-|^2 - |\alpha_k^+|^2) + Q^2 (|\alpha_k^-|^2 - |\alpha_k^+|^2)^2} \right) \quad (3.40)$$

However we also know that the constraints on the a_i and the definition of A mean that:

$$\begin{aligned}
|\alpha_k^-|^2 + |\alpha_k^+|^2 &= \sum_{i=1}^n a_i = \frac{7}{3} \\
|\alpha_k^-|^2 - |\alpha_k^+|^2 &= -\sum_{i=1}^n a_i m_i = -A
\end{aligned} \quad (3.41)$$

Thus we get the final two eigenvalues as:

$$\lambda^\pm = \frac{1}{2} \left(\frac{7}{3}Q + 2S + T \pm \sqrt{\frac{49}{9}Q^2 - 2AQT + T^2} \right) \quad (3.42)$$

Although the final expressions for the eigenvalues of the η matrix are somewhat unwieldy when expressed in terms of A it can be shown that they are monotonic functions in the range $-\frac{7}{3}$ to $-\frac{1}{3}$. Plots of these eigenvalues can be seen in figure 1. The third graph — corresponding to the λ^- eigenvalue — is the only one which gives negative eigenvalues. It is clear from reading off the boundary values that the magnitude of the eigenvalue is bounded by 4. Note, however, that this eigenvalue is undefined at $A = -\frac{1}{3}$ so we cannot saturate the bound.

It can also be seen that if we take, in particular, the seven moduli case with all weights $a_i = 1/3$ and take two of the fluxes to be positive, (thus setting $A = -1$) we find the following spectrum of m^2/V_0 : 10.90, 6.12, 2.45, -6.76. None of these are integer thus providing a case of non-integral scalar masses in a dS compactification.

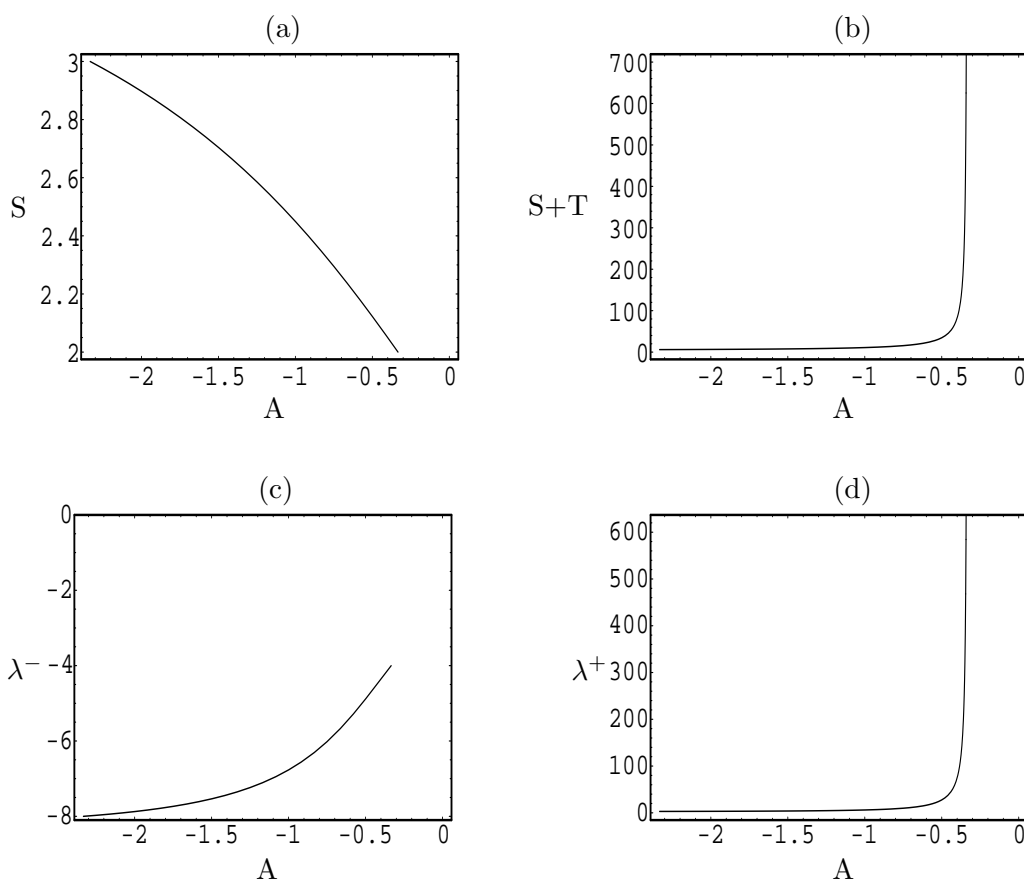


Figure 1: The plots show the eigenvalues of the η -matrix for all permissible values of A ($-7/3$ to $-1/3$). From the top left, going clockwise, we have S , $S + T$, λ^+ and λ^- .

4. Applications to cosmology

At first blush it appears that there are no particularly useful applications to cosmology since in all cases the tachyonic directions are rather steep (numerically they seem to be close to the cases studied in [2]). Furthermore, since 2002 when such models were considered for dark energy, new data has become available which point us closer towards a cosmological constant (see, for example, [12]). This would mean that we need even flatter potentials than those studied in [2] if we would like to use the dS saddle points to explain the current accelerating expansion of the universe. However, it is worth taking a brief, closer look at the general picture of this tachyonic “fast roll inflation” ([13]). We will follow the treatment in [13] closely. Note that we are working in Planck units, with the $4d$ Planck mass set equal to 1.

Near the unstable de Sitter point we can model a generic tachyonic potential as:

$$V(\phi) = V_0 - \frac{m^2 \phi^2}{2} \tag{4.1}$$

V_0 will of course be given by the value of the potential at the saddle point and m^2 will depend on the value of the a_i , but (as demonstrated above) will lie between $8V_0$ and $4V_0$.

Defining ϕ_* as the point at which the potential reaches half its maximum value, we have:

$$\phi_* = \frac{\sqrt{V_0}}{m} \tag{4.2}$$

The Hubble constant, H , remains fairly constant (at $H^2 = V/3$) while ϕ is in the range $\phi_0 < \phi < \phi_*$ and it can be shown [13] that the total expansion of the universe in this period is given by:

$$\frac{a(t_*)}{a_0} \approx e^{Ht_*} = \left(\frac{\phi_*}{\phi_0}\right)^{1/F} \tag{4.3}$$

F is given by:

$$F\left(\frac{m^2}{H^2}\right) = \sqrt{\frac{9}{4} + \frac{m^2}{H^2}} - \frac{3}{2} \tag{4.4}$$

It is immediately clear that irrespective of H and m one could achieve an arbitrary number of e-foldings by making ϕ_0 small. However, our ability to do this is constrained by the effects of quantum fluctuations of the ϕ field. In [13] this constraint is calculated to give a minimum value for ϕ_0 of m/C , where $C = O(10)$, which in turn means:

$$e^{Ht_*} \sim \left(\frac{10\phi_*}{m}\right)^{1/F} \tag{4.5}$$

For our model $\phi_* \sim O(1)$ (in Planck units), so:

$$e^{Ht_*} \sim \left(\frac{10}{m}\right)^{1/F} \tag{4.6}$$

Working with the steepest case gives $F^{-1}(8/3) = 0.72$. To calculate an estimate for the number of e-foldings we now need an estimate for m .

Clearly, calculating m is equivalent to calculating V_0 (since $m^2 = 8V_0$), and we have the following expression for V_0 (from 3.2):

$$V = \frac{c_2^2}{48\pi V_X^3} \left(3 + \sum_{j=1}^n a_j h_j (h_j - 3)\right) \tag{4.7}$$

Recall that V_X is given by 2.5:

$$V_X(s) = \prod_{k=1}^n \left(\frac{h_k}{\nu_k}\right)^{a_k} = \prod_{k=1}^n \left(\frac{-c_2 a_k h_k}{N_k}\right)^{a_k} \tag{4.8}$$

Continuing to look at the steepest case ($\eta = -8$) we can use our earlier results (along with the observation that for this de Sitter all the fluxes are negative and consequently $A = -7/3$) to get $h_j = 3$ and thus:

$$V_0 = \frac{c_2^2}{16\pi V_X^3} \quad \text{with,} \quad V_X = \prod_{k=1}^n \left(\frac{-3c_2 a_k}{N_k}\right)^{a_k} \tag{4.9}$$

Since this is dependent on fluxes, it is manifestly tunable, and thus we are (though with some limitations) free to chose V_0 . Before doing so, however, it is worth considering this tuning a little more carefully. One straightforwardly sees that to achieve a small V_0 we need to make the volume, V_X , large. For volume we need the fluxes to be as small as possible (i.e. $O(1)$ integers). Then, assuming the a_i are all of similar order we will have $V_X \sim c_2^{7/3}$ (and similarly the compactification scale will go as $R \sim c_2^{1/3}$), thus:

$$V_0 \sim c_2^{-5} \tag{4.10}$$

So, to obtain $V_0 \sim 10^{-120}$ we need $c_2 \sim 10^{24}$. Although there is no known explicit construction of a G_2 manifold with such a c_2 , it does not appear that such a thing should be prohibited. We note, though, that whilst these large volume manifolds may exist they will only account for a small fraction of the total number of G_2 flux compactifications [1].

Now, $c_2 \sim 10^{24}$ implies that $V_X \sim 10^{56}$ and that $R \sim 10^8$. As required, note that since there are still many orders of magnitude difference between the size of the internal space and the dS space, we should still be within the regime where the supergravity approximation is valid. For a further consistency check we should try and find the corresponding scale of Kaluza Klein excitations. To do this we first need the $11d$ Planck mass. The calculation of m_{kk} follows [7].¹ This is given, in the usual fashion, by:

$$m_p^2 \sim V_X M_p^2 \sim c_2^{7/3} M_p^2 \sim 10^{56} M_p^2 \tag{4.11}$$

m_p and M_p are the $4d$ and $11d$ Planck masses respectively. Then the KK scale will be given by:

$$m_{kk} \sim \frac{M_p}{R} \sim \left(\frac{1}{c_2}\right)^{\frac{1}{3}} M_p \sim \left(\frac{1}{c_2}\right)^{\frac{3}{2}} m_p \sim 10^{-36} m_p \sim 10^{-17} \text{GeV} \tag{4.12}$$

While this is evidently extremely low compared to not only the Planck scale, but also to any experimental bound, this alone is not enough to mean that we should reject the model. As discussed in [7] the KK modes should transform under $7d$ gauge transformations, whereas the standard model fields transform under $4d$ gauge transformations. Accordingly, it would be difficult to construct renormalizable interactions between the two and hence the KK modes would be hard to detect.

Returning to the central calculation of this section, the final step is to set V_0 equal to the observed value of the cosmological constant ($10^{-120} M_p$). This in turn gives $m \sim 10^{-60}$ and thus:

$$e^{Ht_*} \sim (10^{61})^{0.72} \sim 10^{44} \sim e^{100} \tag{4.13}$$

Of course, the universe can inflate e^{100} times only if the field ϕ initially was very close to the point $\phi = 0$. However, to describe the present stage of expansion of the universe we

¹Acharya obtains a very different value m_{kk} , since the chosen value of c_2 is much lower. There the aim is to ensure that the fundamental Planck scale is the same as the weak scale. Here, on the other hand, we are attempting to address the cosmological constant problem, not the hierarchy problem, hence the much larger value for c_2 .

only need to have one or two e-folds of accelerated expansion. A numerical investigation of this issue shows that this can be achieved if the initial value of the field ϕ is few times smaller than the Planck mass. Such initial conditions seem quite natural, especially if one takes into account that for ϕ a few times greater than M_p the universe rapidly collapses and cannot support life as we know it.

This fact was the basis for an anthropic solution of the cosmological constant problem in the models of a similar type in [3]. All results obtained in [3] should remain valid for our class of models, up to coefficients $O(1)$. But in our case in addition to the anthropic constraints $V_0 \lesssim 10^{-120}$ and $\phi \lesssim M_p$ we also have a related anthropic constraint $c_2 \gtrsim 10^{24}$. It remains to be seen whether the models with such enormously large value of c_2 exist, or if this requirement rules out the class of models proposed in [1].

Furthermore, the combination of this requirement to have a large c_2 , the relatively sensitive initial conditions and the absence of light KK-Standard Model interactions begins to have the appearance of a tower of “ifs”. While this does not preclude us from considering this class of models for cosmology, it does make them somewhat less than desirable.

5. Conclusions

We have explored the consequences of the models of [1] with specific reference to the kind of cosmologies they can give rise to. While it appears that such models do not allow slow roll regimes, it may be the case that the tachyonic directions of the moduli potential can give rise to fast roll inflation, which in turn can provide a mechanism for the current accelerated expansion of the universe. However, in order to achieve the kind of acceleration seen today we are forced to only consider manifolds where the topological invariant c_2 takes large values. This may seem somewhat unnatural, but there appears to be no a priori reason that such manifolds should not exist and could not be constructed.

As a final point, it is worth noting that whilst this set of models does not appear to give rise to the full tapestry of cosmologies that may describe our universe, that does not necessarily deprive the analysis of value. The landscape of vacua is vast and so the possibility of excluding sections of it as unusable is, in some ways, as useful as adding new models.

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